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GEOMETRIC PARAMETERS OF THE CLOCK DIAL

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Abstract This paper deals with the geometric parameters required for the construction of a well-designed clock face as a part of pendulum wall clock described thoroughly in the master work. The face or dial of a clock is a circle whose circumference is divided into 60 equal parts, called minute spaces. A clock has two hands, the smaller one is called the hour hand or short hand while the larger one is called the minute hand or long hand. In order to determine the relation between the diameter of the clock and the distance from which one can clearly read position of its hands, it is necessary to adopt some of the basic geometric characteristics of minute and hour division on the clock face. Second important thing when it comes to the construction of a dial, is the height of the clock face. The criteria for dial height can be derived from the approved standards for the view field width of increased attention.

Keywords: Clock; mechanisms; clock face geometry; clock hands; pendulum.

1. INTRODUCTION

To make the clock usable, i.e. fit its purpose, it is necessary that the geometric parameters of its dial be consistent with certain requirements and recommendations. These requirements and recommendations derive directly from the physiological characteristics of normal human eyesight that today represent ophthalmic standards.

The first standard to be used in this simple calculation is the so-called visual acuity of the naked eye and is defined as the resolution of the eye or the visual power of detail separation. The average or adopted "normal" resolution of the human eye is about 1 angular minute. It is statistically determined and means that the "average" eye can distinguish between two contours if their distance is viewed at a viewing angle equal to or greater than one arc minute. Also, it means that the detail (point, line) is noticed and distinguished from the surrounding details if viewed at an angle of not less than one arc minute. If this angle is smaller, the "average" eye does not recognize the two contours, i.e. the details as separate.

Another standard that will be used in formulating recommendations for the geometric parameters of the dial refers to the width of the so-called field of vision of the heightened attention of the human eye. The width of the field of vision of the human eye is quite large: $\pm 110^0$ in the horizontal plane and $\pm 60^0$ in the vertical plane, measured from the optical axis of the eye. However, the observation of shapes, details, colours and movements is not evenly distributed over the entire width of the field of vision, which is a direct consequence of the uneven density of optical sensors on the retina of the eye. The density of these sensors (the so-called cones and rods) is the highest on the macula of retina and decreases sharply with increasing radial distance from macula. Characters that fall on macula are the focus of the beholder's attention, and attention diminishes as the spotted characters are further away from it. It is adopted as a standard that the field of vision of increased attention lies within a

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rotating vision field of cone whose derivatives close a 30 degree angle with the optical axis of the vision cone. If the observed objects are within this cone, the observer will notice them with increased attention. If it is outside, the observer may notice the object, but unconsciously, so they will not pay attention to it.

2. GEOMETRY AND VISUAL CHARACTERISTICS OF THE CLOCK DIAL

In addition, it is necessary to highlight some recommendations regarding the geometry of the dial itself. It is advisable for the dial to be circular and not any other (triangular, square, rectangular ...) shape, because only the circular shape ensures the uniformity of the hour and minute markings and their equal distance from the center of the clock dial.

The clock dial should have 12 hour markings and 60 minute markings [1]. Divisions can be realized as points or lines of appropriate dimensions. The lengths of the divisions into 12 may be all the same, but for clarity the division into 4 can be highlighted (12, 3, 6, and 9 hours) by the increased length of the lines. The size (length and width) of the 60-minute dividing lines must be at least twice the size smaller than the 12-hour dividing lines.

The clock must have a clock and a minute hand, and a secondary if needed [2]. The clock hand should always be shorter than a minute one, approximately equal to one quarter of the diameter of the clock and a thickness equal to or greater than the thickness of the hour markings. The length of the minute hand must not be smaller than 3/4 nor greater than 5/6 of the radius of the clock, and a thickness of at least equal to the thickness of the minute markings. The secondary hand, if any, should be the thinnest and longest, but highlighted with a specially selected colour, marking, or shape. These recommendations are of a practical nature: the observer should first notice the hours and, if able, because of the distance, the minutes and finally the seconds.

In accordance with the aim of the observer to read the time on the dial of the clock effortlessly, precisely and quickly, it is necessary to ensure a sufficiently high contrast of markings. It has been experimentally determined that the greatest contrast is provided by the combination of black and yellow with the black background of the dial and the yellow division markings. Somewhat less contrast is created by the black background and white markings [3]. The combination of a black marking on a white background, although widespread in use, causes the eye of the viewer to be dazzling, thus not contributing to the clarity of time reading. If, for aesthetic or any other reasons, a black background is not acceptable, dark blue with yellow or light orange markings may be used. Also, dark green or even dark red (burgundy), with white or yellow markings for hours and minutes, may be adopted for the background colour of the dial. All colours should be matte in order to prevent reflections that impair the clarity of reading of the hands positions.

Public clocks should be placed in illuminated places, not shaded or obscured by other objects [1], [4]. It is important that they be adequately lit at night, with special lighting or pre-existing street lamps. If the dial is white and the markings are black, it can be made of matt glass, which allows it to be illuminated at night by the light sources behind the dial. The light should never be too strong, dazzling or of colour that is not white or light yellow. The public clock dial should not be glazed, protected by a glass window, because of reflections.

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Numbered markings, although regularly present on many public clocks and wristwatches, are not necessary. Namely, the division into 12 ordinary lines (or circles - dots) achieves an unambiguous orientation on the dial so that the observer will be able to observe and read clearly and infallibly the positions of hour and minute hands without numbers as well. The following example illustrates that the numbered markings are not necessary. If the minute hand points to 3, it reads "... and fifteen", if it shows 4, we say "... and twenty", if it is at 6, we say "half", if it stands at 8, we read "twenty to ...", if the minute hand shows 9, we say "fifteen to ..." etc. Thus, the numbered markings may be present, but they are not necessary because they are not read, but the geometric division of the circle into quarters, thirds, and halves.

In order to establish the relationship between the diameter of the clock and the distance from which the position of its hands can be read clearly, it is necessary to first adopt some basic geometric characteristics of the minute and hour divisions on the dial. To distinguish the hour division from the minute one it is sufficient to assume that the thickness of the hour division line is twice the thickness of the minute line. It is also accepted that the distance. between any two nearest lines on the division of the dial should not be shorter than the thickest line, ie. the thickness of the hour division line.

If the arc distance between the axis of symmetry of any two lines on the division of the dial is divided into seven equal parts and if it is chosen that the thickness of the line of the hour division is 4/7 and minute 2/7, then the recommendations adopted in the previous chapter are met. This division can be expressed in angular measures: the thickness of the line for the hour division corresponds to the central angle of the dial of α =(4 · 360 °)/(420 = 3.43 °), and for the minute division β=α/2= 1.71 °. A smaller and larger division is also possible, but it would be at the expense of resolution or visibility. Figure 4.1 shows the basic geometric characteristics of the minute and hour divisions on the dial. If the diameter of the dial is D, then the thickness of the minute division line is 2Dπ/((7 · 5 · 12)) = 2Dπ⁄420 and, in order to be viewed from a distance l at a viewing angle of one arc minute, the following equality must be applied:

$$\frac{2D\pi}{420} = l \cdot tan\ tan\ \frac{1}{60\ °}$$

The relation follows directly from the above equation and that is

$$\frac{l}{D} = \frac{\pi}{210 \cdot tan\ tan\ \left(\frac{1}{60\ °}\right)} = 51.42 \approx 50.$$

So, if the visual conditions are good (brightness, the air is free of smog and fog), the minute division of the dial of a clock whose diameter is D will be seen clearly and effortlessly at a distance of l = 50D. The clock with the dial of diameter D will be usable under good light conditions and at a distance of 100D, since the lines of the hour division are twice as thick as the minute lines, and no minute reading is required for approximate reading of time. As the lighting conditions are often poor (dusk, dawn, the dial is not illuminated or is in shadow, there is fog or smog in the air, rain, snow, etc.), it is correct for safety reasons to adopt a recommendation that for the maximum reading distance of time l a clock dial of diameter D =l/50 is provided and designed.

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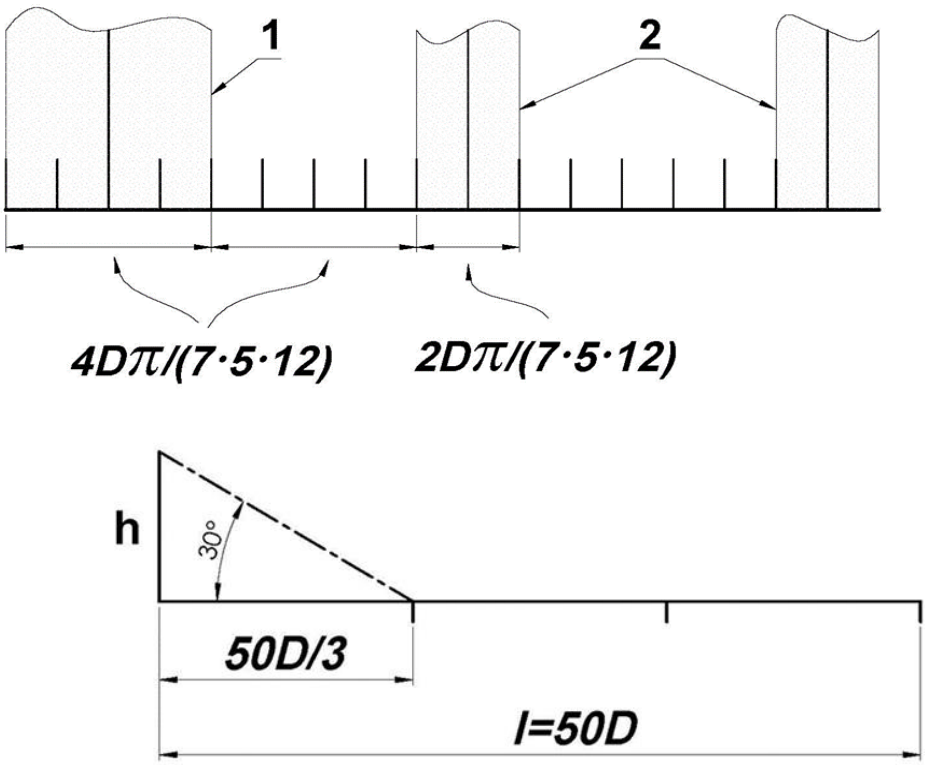

FIGURE 1. Parameters for determining the visibility of the clock.

For the selected diameter of the clock dial D, it is also necessary to establish a recommendation for the height at which the clock (axis of the clock hands) will be set. Ideally, every clock should stand at the height of a human face - at eye level. However, such clocks would rarely be usable as they would be regularly obscured by other numerous objects. Therefore, it is necessary to set the clock to a higher altitude, but again not too high. The height criterion can be derived from the adopted standard for the width of visual field of increased attention.

3. NUMERICAL EXAMPLE

Let us suppose that the clock dial of diameter D has been designed according to the rules already described. A reasonable recommendation for the height of installation h of such a dial is that the dial should be placed at that height h so that, from a third of the length l = 50D, it is visible at the border of the visual field of increased attention. From this recommendation the following expression is derived:

$$\frac{3h}{50D} = tan\ \tan 30\,^{\circ},$$

From which it directly follows that installation height of the dial is

$$h = \frac{50D}{3}\, tan\ tan\ 30\,^{\circ} = 9.62D.$$

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Thus, if the dial of the clock of diameter D is placed at a height h which is 9.62 times bigger than its diameter, it will be viewed at the boundary of the field of vision of increased attention, or at an angle of 30 °, from a third of the distance l = 50D. As the dial is viewed from the surface of one circular sector of radius l = 50D and if raised to the height h = 9,62D, it will be within the visual field of increased attention of the observer with 8/9 of the total area of the said sector. If necessary and if circumstances permit, the dial can be set to a lower altitude but not to a higher altitude.

Public clocks are set up as façade, street clocks or clocks at squares [5]. If the clock is on the facade, it has only one dial, the street ones have two dials that look on both sides of the street, and the clocks at squares should have four dials that are turned in all four cardinal directions.

For example, if the square is approximately circular in diameter of 100 meters, the clock should be placed in the center of the square, it should have four dials of 1 meter in diameter each, rising to a height of 9 or 10 meters. They will be in the visual field of increased attention from an area of almost 7000 of a total of 7854 square meters. Home clocks - alarm clocks, desk clocks, wall clocks, etc., should have dial diameters tailored to the dimensions of the rooms. In most cases, it is sufficient that their diameters are not shorter than 80 to 100 millimeters.

Figure 2. Large facade clock at the beginning of Knez Mihailova Street.

All of these recommendations should be followed in practice because there is no reasonable reason not to do so, except through negligence or ignorance. Also, the stated and explained requirements are sufficiently mild and general, thus providing designers with ample space for the freedom of additional aesthetic expression and artistic creativity. Nonetheless, there are too many examples, especially public clocks, where these recommendations are not respected at all or are hardly met. Thus, for example, a rectangular façade clock at the main post office in Belgrade, approximately one meter in diameter, has been set at a height of about thirty meters. Most passers-by have never even become aware that the clock exists at all. The clock at the top of the Railway Museum building in Nemanjina Street, Belgrade, has also been set disproportionately high for the diameter of its dial. The large façade

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clock at the beginning of Knez Mihailova Street in Belgrade (Figure 2) does not meet most of the recommendations highlighted here. Yet, because of its dimensions and the small height at which it has been set, it would justify the purpose of its existence, had it not been for one but the worst possible mistake: two trees have been planted directly in front of it, as if it had been intended for the clock not to be seen at all during the whole spring and summer. The damage has been made greater due to the fact that this clock has become a kind of symbol of the city of Belgrade, just at a time when there were no treetops in front of it.

4. EXAMPLES OF PUBLIC CLOCKS IN SERBIA

Fortunately, there are also examples in Serbia where public clocks do their part correctly. Thus, for example, the old, large façade-tower clock at the Palace of Justice in Niš has been designed correctly Belgrade (Figure 3). It would be better if the background of the dial was black and the markings were white or yellow. No objection can be made to the ecclesiastical, quadruple dial of the tower clock at the Cathedral in Novi Sad (Figure 5). The big black clock at Petrovaradin Fortress (Figure 4), a symbol of Novi Sad, is also an example of a well-designed public clock. The facade clocks at the old Belgrade train station building were also excellent by all the criteria set out here. Our hope remains that they will not be removed upon completion of the ongoing reconstruction.

There are public clocks in the hallways on each floor at the Faculty of Mechanical Engineering in Belgrade (Figure 6). They are blue with white markings, which is commendable. The serious complaint is that they haven't been working for decades. The only thing worse than that would be that they work, but inaccurately, which luckily is not the case. In all three amphitheaters of the aforementioned faculty, it would be useful to place clocks with dials of diameter not shorter than one meter, at the maximum height that the amphitheater allows.

Figure 3. Large faacade-tower clock at the Palace of Justice in Niš.

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Figure 4. The big black clock at Petrovaradin Fortress.

Figure 5. The tower clock at the Cathedral in Novi Sad.

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Figure 6. **Clock in the hall of the Faculty of Mechanical Engineering, University of Belgrade.**

4. CONCLUSION

This paper deduces, derives and discloses the geometric and other visual parameters required for the construction of a well-designed clock face. In particular, the formulas for the correct clock face diameter, the height of the dial, dimensions of the division marks and clock hands are obtained from two important ophthalmological parameters: visual acuity and field of vision of the heightened attention of the naked human eye. The relationship between the diameter of the clock and the distance from which the position of its hands can be read clearly is also emphasized. In addition, regarding the sufficiently high contrast of clock face markings, the appropriate and applicable combination of colours and illumination of the clock dial parts are also discussed and recommended. Finally, a few examples of public clocks in Serbia are presented and the correctness of their design are considered and criticized.

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