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STRUCTURAL OPTIMIZATION AND MODIFICATION - SHORT REVIEW

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Abstract Investigations in structural optimization pointed to the wide interest in the subject, to the need for a realistic evaluation of possible directions of development, and to the question of the relevance of optimization techniques in the design of engineering structures. The present paper deals with the problem of improving dynamic characteristics of some structures.

Keywords: Structural optimization; reanalysis; structural modification.

1. INTRODUCTION

Historically, in the earliest period of modification of structural variables, most research related to static optimization, such as weight minimization with the necessary restrictions on movement or voltage, for different static load conditions. Two approaches, analytical methods and numerical methods were usually used. For simple problems, analytical construction methods could be usable for geometric optimization. In this sense, mathematical calculations, variation methods, etc. were used. The solutions, however, often did not have practicality, and could not be applied to the real-complicated structures composed of many types of structural elements. However, for the simple problems, solutions could provide some insight into the optimization concepts. As an example of analytical approximate, the optimization criterion is defined. It is used for an iterative procedure for meeting the necessary optimization conditions. The optimization approximate criteria can be effective, but sometimes intuitive. Instead of the silent job to find the necessary conditions for optimization criteria, using numerical methods it is possible to directly minimize a specific function with the required limitations. The rapid development of numerical mathematics has provided many effective iterical procedures, which give approximately optimal values using super fast computers. In these methods, the final number of structural variables are included. It is also important to reduce the number of construction variables, the "weight" problem "depends directly. Mathematical programming technique in combination with the finite element method is a secure indicator that there will be an efficient and direct approximation. If the system becomes great, where the number of degrees of freedom increases, the efficiency of this technique is declining due to budget difficulties and prices. It is natural, so, the field of dynamic modification appeared in the literature. The basic theory for determining the existence of solution for frame structure optimization with frequency limits is found in [1]. According to this theory, natural frequencies do not change with uniform frame modification and key limitation for determination of optimal dynamic solution of frame structure modification is mostly that of eigenfrequencies. The optimization criteria for space frame structure with multiple limitations in its natural frequencies are considered in [2]. Knott coordinates and cross

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sections of elements, although of different nature, have been treated simultaneously in unified design specification for a minimum weight of structure. Optimum first criterion, developed for one limitation based on differentiation of the Lagrange function, indicates that at optimum all the variables are of the same efficiency. In order to solve multiple limitations of frequencies global numbers are introduced, avoiding in this way the calculation of Lagrange's multiplicators. In the final stage, the most efficient variables are identified and modified as priority. Using the minimal weight increment, optimal solution can be obtained from initial design solution. The procedure is also effective for repeated values of frequency. In paper [3], the model for modified dynamic structural system is presented, based on reduced appreciative concept of improved method for the approximation of eigenvalues and eigenvectors of first order. The expressions for local approximation based on Taylor's series are used as base vectors for eigenparameters perturbance approximation. The reduced system of eigenvalues is generated for each eigenvector using eigenvectors as a base and Ritz's vector approximation of first order. A new function to limit eigenvalues approximation in the procedure of structural optimization is introduced in [4]. Applied Rayleigh's ratio increases the approximation quality for frequency limitations since it approximates eigenforms energy and kinetic energy instead eigenvalues, producing faster and stable convergent solutions.

2. PROBLEM STATEMENT

2.1. Structural and characteristic variables

There are two types of variables imposed in the problems of dynamic modification and these are: structural (structural) variables and characteristic variables.

2.1.1. Structural variables (structural parameters - design variables)

Construction variables may refer to changes in structure (geometry), elements and materials. The change in the structure geometry characterizes the change of knot coordinate (boundary conditions), while changes in the size of elements relate usually to the characteristics of transverse cross section. For specific final elements, different physical sizes can be used as structural parameters, for example, the transverse cross section of beams and inertia measurements can be used instead of cross-sectional dimensions. Construction variables can be continuous or discreet. Usually, shape and size are continuous, while the material is a discreet variable. By applying composite materials of high quality constructions, it can "be strengthened" and "ease". The fact is that discrete structural variables make modification problems very complicated and expensive. In current issues, structural variables are usually imposed on a group of elements, and less often elements individually. In other words, the characteristics of several elements are changed together in the same relationship, which leads to improving the desired performance in operation. Thus, the number of variables is much smaller than the number of modified elements, which are suitable for the modification procedure. This approximate is called - connecting structural variables. Geometric changes that do not apply to a simple change in model size are very complex and expensive because all coordinates on the structure network must be redefined. The most favorable elements are simple elements of a stick type, beam or plate.

2023, Volume 7, Issue 1, 11-17, DOI 10.6723/TERP.202303_7(1).0002

2.1.2. Characteristic variables

In dynamic optimization problems, the term "characteristics" refers to frequencies (own values), main forms of oscillation (own vectors) or dynamic response. Constraints with respect to frequency values are given by appropriate inequalities. If modal changes are not so critical then restrictions in mind are suitable equations. The numerical values of their own vectors do not contain absolute intensities in itself, so relative amplitudes should be compared to some reference values. To this end, the method of normalization of their own vectors on unit value for the largest component or unit value for a specific component is better than normalization in relation to ground. Although the method of normalization should not be a critical problem, Young and Christiansen [5], indicate that instability can be instability depending on the normalization approach. The specification of all degrees of freedom in "modified" modal forms or is possible or desirable in large structural systems. Too many restrictions in "perturbated" modal forms can deprive the system of opportunities to achieve minimum objective functions to their own regulation. For large systems, methods are needed to reduce the number of variables. Guyan's static reducing scheme does not seem to make sure the results. The usable dynamic reduction is generally generated in a dynamic reduction method, which represents a combination of Guyan reduction and iteration of subspaces. The method of eliminating unwanted degrees of freedom in static problems was elaborated in [6]. A similar procedure can be applied in a dynamic analysis [2]. For some given degree of freedom R, determining their own values and own vectors is much more expensive than in static problems. It is also desirable to limit degrees of freedom in discretized systems to make them more economical. For example, it is generally a practice that only degrees of freedom are taken into account that represent a medium level in say the aircraft wings (although they are shifted in case of static analysis). The general method for condensation of the degree of freedom, is not limited to dynamic analysis but represents a summary technique for reducing the number of equations of systems arising from the final structural analysis.

2.1.3. Sensitivity of own values and own vectors

One of the most popular methods of realization is an analysis of sensitivity that is successfully applied in general and specific dynamic problems. In this field, the sensitivity of own values and own vectors and first-order was developed with an aspect to predict the dynamic response of a modified structure from knowledge of its spatial and modal properties in the original, unmodified condition. As the analysis of the sensitivity of a mechanical structure based on developing Taylor's own values and its own vectors of unmodified structure, the calculation of higher order members is difficult and time-dabic, so the efficiency of this model is limited by small modifications. However, it is not easy to determine what "little" means. The analysis of sensitivity is extremely important in the construction of construction when known modal properties, either from theoretical or experimental analysis. The sensitivity of modal characteristics is the conduct of modal characteristics of the dynamic system in relation to selected construction variables. There are two primary applications of sensitivity analysis in the literature that deals with this issue. In the first case, data on sensitivity is only taken as a qualitative indicator of the location and approximate scope of structural changes to achieve the desired changes of dynamic traits. The consequence of this approach is that data on changes in structure are obtained using accurate methods. In the second case, sensitivity of modal properties is directly used to predict the effects of proposed or desired structural changes. The use of sensitivity in this way relies on the Matrix Taylor's rows, with the usual convergence expectations as

2023, Volume 7, Issue 1, 11-17, DOI 10.6723/TERP.202303_7(1).0002

well as the mistakes due to the neglect of higher order members. Using the only first-order members involves implicitly neglecting others and higher excerpts. If members of second-order are used for analysis, this can be a convenient criterion for assessing the admissibility of the first order in the analysis of prediction in some detail. The analysis of sensitivity can be applied partly if the changes and functions of characteristic variables are known depending on modification parameters, but it is certainly limited to parts of the structure. Modal sensitivity can also be defined as a statement of one's own equation of the dynamic system at those variables that are suitable for modification. A typical modification can be a change in the diameter of the circular cross section. This can also affect the mass proportional square in diameter, and stiffness, which depends on the axial moment of inertia crossing. The change in length has a proportional impact on the change in mass and the cubic impact on the stiffness. Changing the type of material (mechanical characteristics) has proportional impact on the change of mass and a change in stiffness.

The matrix form of the differential equations of motion of the system represented by the FE model, in the case when there are no external forces:

$$[M]\cdot\left\{\underset{\sim}{\ddot{Q}(t)}\right\}+[K]\cdot\left\{\underset{\sim}{Q(t)}\right\}=\{0\} \tag{1}$$

where [M] and [K] represent mass and stiffness matrices, respectively. The formula for the sensitivity of the eigenvalue λ at the i-th mode for the j-th modification parameter p, from which it can be seen that the sensitivity of the eigenvalue for the variable p_j can be calculated from the eigenvalue, the corresponding parameter, and the sensitivity of the mass [M] and stiffness [K] matrices for the modification parameter.

$$\frac{\partial\lambda_i}{\partial p_j}=\{\underset{\sim}{Q_i}\}^T\cdot\left(\frac{\partial[K]}{\partial p_j}-\lambda_i\cdot\frac{\partial[M]}{\partial p_j}\right)\cdot\{\underset{\sim}{Q_i}\} \tag{2}$$

The eigenvector ({Q_i}) sensitivity equation is given below:

$$([K]-\lambda_i\cdot[M])\cdot\frac{\partial\{\underset{\sim}{Q_i}\}}{\partial p_j}=\frac{\partial\lambda_i}{\partial p_j}\cdot[M]\cdot\{\underset{\sim}{Q_i}\}-\left(\frac{\partial[K]}{\partial p_j}-\lambda_i\cdot\frac{\partial[M]}{\partial p_j}\right)\cdot\{\underset{\sim}{Q_i}\} \tag{3}$$

If the i-th mode is not a double mode, there is only one eigenvalue that is equal to λ_i, among all the eigenvalues of equation (1). In this case, the solution of equation (3) can be obtained as shown in [7]. It is assumed that l is the position of the element with the maximum absolute value of the eigenvector {Q_i}. Let all elements in the l-th row and l-th column of the matrix of coefficients in equation (3) be equal to zero, but let the diagonal element in the l-th row be equal to unity. The corresponding l-th element on the right-hand side of equation (3) is also equal to zero. Then an equation with a non-singular matrix of coefficients was set

2023, Volume 7, Issue 1, 11-17, DOI 10.6723/TERP.202303_7(1).0002

$$l \rightarrow \begin{bmatrix} k_{11}-\lambda_i m_{11} & 0 & k_{13}-\lambda_i m_{13} \\ 0 & 1 & 0 \\ k_{31}-\lambda_i m_{31} & 0 & k_{33}-\lambda_i m_{33} \end{bmatrix} \cdot \{\xi_{ij}\} = \begin{Bmatrix} b_1 \\ 0 \\ b_3 \end{Bmatrix} \leftarrow l \tag{4}$$

The solution obtained from the above equation can be considered as one of the solutions of equation (3). Therefore, any solution of equation (3) can be written in the form

$$\frac{\partial\{Q_i\}}{\partial p_j} = \{\xi_{ij}\} + \{Q_i\} \cdot c_{ij} \tag{5}$$

Coefficient c_{ij} can be obtained from the following expression:

$$c_{ij} = -\left(\{Q_i\}^T \cdot [M] \cdot \{\xi_{ij}\} + \frac{1}{2} \cdot \{Q_i\}^T \cdot \frac{\partial[M]}{\partial p_j} \cdot \{Q_i\}\right) \tag{5}$$

With c_{ij} from the above expression and $\{\xi_{ij}\}$ from expression (5), the sensitivity of the eigenvector of the i-th mode with respect to the modification parameter p_j can be calculated from equation (3). Compared to the expressions for the sensitivity of the eigenvalues, the calculation of the sensitivity of the eigenvectors is much more complicated and takes much more time. To solve the equation (3), the computer time depends on the number of degrees of freedom of the KE model. From equation (3), it is also possible to see that for the sensitivity of the eigenvectors from the main forms to all the modification parameters, the coefficient matrix of the equation will remain unchanged. If the sensitivity of the main vector is calculated for the modification parameters - one by one, the equation must be solved several times even though the matrix of coefficients is the same. When there are *m* modes in the modification object and *l* modification parameters, this equation needs to be solved $l \times m$ times for each principal mode of oscillation and each modification parameter. Following the same procedure described above, a matrix can be obtained where each column represents the sensitivity of its own vector with respect to some modification parameter. In this way, solving a linear equation with a matrix of high-order coefficients is done only once for one eigenvector. The sensitivity matrix with respect to the unspecified parameters is used to select the reference modes and degrees of freedom that should be modified. Examining the sensitivity on real examples is still a rather messy job, so instead the analysis of the distribution of certain quantities is done. The distribution of optimization elements represents a reanalysis expressed in percentages of individual sizes per selected group of elements. Setting the task of obtaining the desired first or some other natural frequency of the system, in the analysis of structures it is often necessary to calculate a large number of structural variants. However, through reanalysis based on balancing the distribution of kinetic and potential energy of all finite elements of the model, it is possible to effectively obtain the desired natural frequencies of the system.

2.2. Example

On the example of a simple structure composed of three hinged rods, the analysis of energy distribution and corresponding conclusions will be shown.

2023, Volume 7, Issue 1, 11-17, DOI 10.6723/TERP.202303_7(1).0002

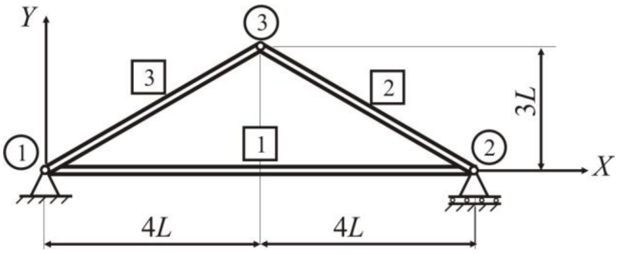
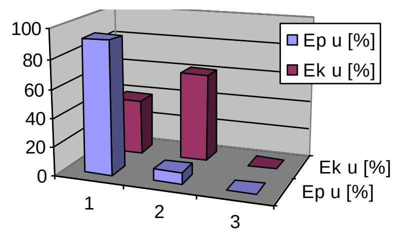


Figure 1. Simple three rods structure.

Analysis of the dynamic behavior of a simple structure is analogous to the analysis of the behavior of a complicated structure. On the example of three rods, the following can be observed. If the kinetic and potential energy of some oscillation mode are equal to zero, there is no need for modification. If the kinetic energy is significantly greater than the potential and vice versa, modification is required.

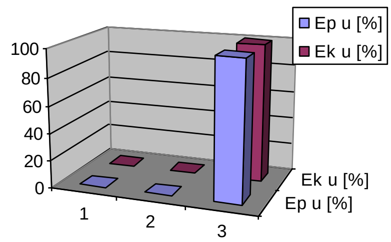
Table 1. Distribution of kinetic and potential energy.

	First mode shape		Second mode shape		Third mode shape	
	E_p [J][1]	E_k [J]	E_p [J]	E_k [J]	E_p [J]	E_k [J]
I rod	0.2982	0.1254	0	0	0.0935	0.4378
II rod	0.0265	0.1992	0	0	1.0507	0.7062
III rod	0	0	0.5123	0.5123	0	0
$\sum_{i=1}^{3} E_{p(i)}, \sum_{i=1}^{3} E_{k(i)}$	0.3247	0.3247	0.5123	0.5123	1.1442	1.1442



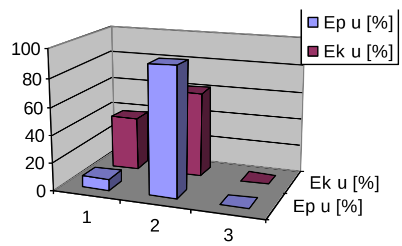
	1	2	3
Ep u [%]	91.8386	8.1614	0
Ek u [%]	38.6322	61.3678	0

Figure 2. Distribution of kinetic and potential energy by given construction on the I form of oscillation in percentage.



	1	2	3
Ep u [%]	0	0	100
Ek u [%]	0	0	100

Figure 3. Distribution of kinetic and potential energy by given construction on the II form of oscillation in percentage.



	1	2	3
Ep u [%]	8.1716	91.8284	0
Ek u [%]	38.2692	61.7308	0

Figure 4. Distribution of kinetic and potential energy by given construction on the III form of oscillation in percentage.

[1] All energy terms are multiplied by a factor $\frac{EA}{L}$. The dimension for energy is [J]. The numbers in the table have a dimension [m^2]

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3. CONCLUSION

Studying the dynamic behavior of structures can be predicted responses due to changes in its shape, size or design elements change materials. The main goal of dynamic optimization is to increase natural frequencies and to increase the difference between them. Especially, the lowest frequencies are the most interesting and those whose values are close to frequency excitation force in the system. Consequently these are further characteristic areas: (a) The elements with kinetics and potential energy, which values are negligible compared to other elements, (b) Elements with the kinetics energy greater than the potential energy, (c) Elements with the potential energy greater than kinetics, (d) Elements with kinetics and potential energy, which values are not negligible compared to other elements. By observing diagrams of the distribution difference of the increment potential and kinetic energy on the modes shape of interest, modification can be suggested. The application of mentioned procedure of real structures shows its practical side.

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