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DESIGN OF FRACTIONAL - ORDER PI CONTROLLER FOR MULTIVARIABLE PROCESS

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Abstract This article presents design and analysis of fractional order PI controller for a pilot plant binary distillation column. Design of controller for a multivariable (multi-input multi-output – MIMO) process is a challenging task due to loop interaction and system with dead time. The first order model with dead time (FOPDT) model is obtained for the overall open loop transfer function of the system. This work aims on the comparative study of one conventional controller along with fractional order controller based on the performance measures for a pilot plant binary distillation column.

Keywords: Decentralized controller; MIMO process; Fractional - order PI controller.

1. INTRODUCTION

In most of the chemical industries the process are multiple input and multiple output process. The most important feature of the multivariable systems is the interactions between its variables or cross couplings. Therefore, systems with multiple actuating control inputs and process outputs are defined as multi-input multi-output (MIMO) systems. Due to the presence of the loop interactions in the multivariable system, the closed loop control system designed should be strong and efficient. The interactions that are due to the change in one input affect many output variables. The appropriate pairing of input and output using suitable loop pairing techniques could minimize the adverse interactions. There are a lot of methods for controller design. Fractional - order PID controllers are very useful one, because it gives more possible sets of controller parameters between an integer degrees of controller terms [1-5]. Various tuning rules are given in [6,7]. Many researchers deal with already known methods for PID controller design in combination with fractional-order ones [8-10]. Distillation is a process in which a liquid or vapor mixture of two or more substances is separated into its component fractions with desired purity. In multi loop control, the MIMO processes are treated as a collection of multi single loops and a controller is designed and implementd on each loop by taking loop interaction into account. For the MIMO processes with severe loop interactions, the decoupling control schemes are often preferred. Due to that, in recent years numerous investigations aim to develop fractional order PID controller for MIMO process [11-17].

The paper is organized as follows. After Introduction in this section, Section 2 gives design methodology for fractional order PID controller and Gain margin-phase margin controller design methodology. Section 3 represents the design and simulation of PID controller, i.e. its shorter variant

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PI controller, for lab scale interacting distillation process followed by the simulation and comparative analysis and Conclusion in Section 4.

2. DESIGN METHODOLOGY

2.1. Fractional - Order PID Controller Design

The work of fractional order proportional integral derivative (FOPID) controller is same as that of the PID controller however it gives better response than the PID controller as it have five tuning parameters such as K_P, K_i, K_d, λ and μ, where K_P is the proportional gain, K_i is the Integral gain and K_d is the derivative gain, λ is integration fractional-order and μ is differential fractional-order for the FOPID controller.The fractional order differentiator can be denoted by a general fundamental operator $a^{D_t^q}$ as a generalization of the differential and integral operators, which is defined in (1).

$$a^{D_t^q} = \begin{cases} \frac{d^q}{dt^q} & ,R(q) > 0 \\ 1 & ,R(q) = 0 \\ \int_a^t (d\tau)^{-q} & ,R(q) < 0 \end{cases} \tag{1}$$

Where q is the fractional order which can be complex number, the constant a is related to the initial conditions. Minimization technique is used to determine the optimum settings of the fractional integration order λ and the fractional differentiation order μ of the $PI^{\lambda} D^{\mu}$ controller. The Integral of Square Error (ISE) is given ba (2).

$$J = \int_0^{\infty} [e(t)]^2 dt = \int_0^{\infty} [r(t) - Y(t)]^2 dt \tag{2}$$

Where $e(t)$ is the error signal, $r(t)$ is reference value and $Y(t)$ is process output.

The equation of the FOPID in Laplace domain is given by (3).

$$G_c(s) = K_P + \frac{K_i}{s^{\lambda}} + K_d s^{\mu} \tag{3}$$

A usually specified control requirement is also to minimise the Integral of Absolute Error (IAE). The Approximate M-constrained Integral Gain Optimisation (AMIGO) method [18] is used to determine the controller parameters because it is suitable for first order plus dead time (FOPDT) models, which very well describe most of processes in industry. Proportional and integral parametres are given with (4) and (5), where K is process gain, T is process time constant, L is process dead time and T_i is integral time constant.

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$$K_p = \frac{1}{K}\left(0.2 + 0.45\ \frac{T}{L}\right) \tag{4}$$

$$T_i = \left(\frac{0.4L + 0.8T}{L + 0.1T}\right)L \tag{5}$$

2.2. Gain Margin - Phase Margin Controller Design

The gain and phase margins are important measures of robustness, associated with frequency response. It is also know from classical control that phase margin is related to the damping of the system, and can therefore also serve as a performance measure. Accurate and simple analytical formulae are derived to tune/design the proportional integral controller for the commonly used first-order plus dead time plant model to meet gain margin and phase margin specifications. Let $G_p(s)$ and $G_c(s)$ denote the process and controller transfer function respectively. From the basic definitions of gain margin and phase margin, following are the set of equations obtained (6-9).

$$arg[G_{pii}(j\omega_{pii})G_{cii}(j\omega_{pii})] = -\pi \tag{6}$$

$$A_{mii} = \frac{1}{|G_{pii}(j\omega_{pii})G_{cii}(j\omega_{pii})|} \tag{7}$$

$$|G_{pii}(j\omega_{gii})G_{cii}(j\omega_{gii})| = 1 \tag{8}$$

$$\varphi_{gii} = arg[G_{pii}(j\omega_{gii})G_{cii}(j\omega_{gii})] + \pi \tag{9}$$

Where A_{mii} and φ_{gii} are desired gain and phase margin, ω_{gii} and ω_{pii} $(i = 1, 2)$ are gain and phase cross over frequencies, respectively. The PI controller is given by (10).

$$G_c(s) = K_c\left(1 + \frac{1}{sT_i}\right) \tag{10}$$

The PI tuning parameters are given by (11), (12) and (13).

$$K_c = \frac{\omega_p \tau}{A_m k_p} \tag{11}$$

$$T_i = \left(2\omega_p - \frac{4{\omega_p}^2\theta}{\pi} + \frac{1}{\tau}\right)^{-1} \tag{12}$$

$$\omega_p = \frac{A_m\varphi_m + \frac{1}{2}\pi(A_m - 1)}{({A_m}^2 - 1)\theta} \tag{13}$$

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3. SIMULATION RESULTS

3.1. WB Model

Wood and Berry (WB) model of distillation column [19] is considered for the simulation and comparison purposes. The process transfer function model is given by (14), and decentralized controller has been designed for it.

$$\begin{bmatrix} X_D(s) \\ X_B(s) \end{bmatrix} = \begin{bmatrix} \dfrac{12.8e^{-s}}{16.7s+1} & \dfrac{-18.9e^{-3s}}{21s+1} \\ \dfrac{6.6e^{-7s}}{10.9s+1} & \dfrac{-19.4e^{-3s}}{14.4s+1} \end{bmatrix} \begin{bmatrix} R(s) \\ S(s) \end{bmatrix} \tag{14}$$

Where $X_D(s)$ it the distillate and $X_B(s)$ is water. The inputs $R(s)$ and $S(s)$ are the reflux flow rate and steam flow rate, respectively. The process has the off-diagonal elements has no right half plane (RHP) poles and diagonal elements has no RHP zeros then the decoupler matrix is obtained as (15).

$$D(s) = \begin{bmatrix} 1 & \dfrac{315.63s+18.9}{268.8s+12.8}e^{-2s} \\ \dfrac{95.04s+6.6}{211.46s+19.4}e^{-4s} & 1 \end{bmatrix} \tag{15}$$

Considering the loop interactions the open loop transfer function of WB model is obtained in the form of FOPDT model (16) and (17).

$$q_1 = \frac{6.37e^{-1.36s}}{5.19s+1} \tag{16}$$

$$q_2 = \frac{-9.65e^{-3.49s}}{4.25s+1} \tag{17}$$

3.2. Simulation Results

Having in mind that in the most cases FOPDT processes are very good controlled with PI controller, in this research this shorter kind of controller has been designed. It is important to say that AMIGO based FOPID controller is designed with integration order λ=1.2, that has been determined using simulations. Table 1. shows tuning parameters for PI controllers for both loops of WB distilation collumn from two different methods.

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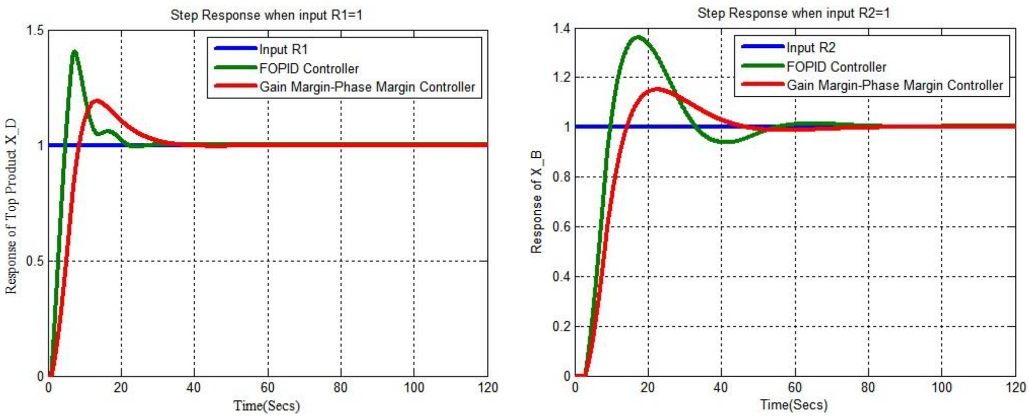
Table 1. PI Tuning parameters for two controller design methods.

Process model	Controller design method	Values of controller parameters	
		K_P	K_i
Wood – Berry model	AMIGO based FOPID controller	Loop 1: 0.30 Loop 2: -0.0775	Loop 1: 0.088 Loop 2: -0.0181
	Gain Margin – Phase Margin method	Loop 1: 0.0784 Loop 2: -0.0380	Loop 1: 0.0554 Loop 2: -0.0174

Figure 1. shows the simulation stuides for the obtained controller with good tracking of servo response.



a) b)

Figure 1. Servo responses a) Output XD, b) Output XB.

After performing of simulations, analysis of performance indices such as IAE, ISE, ITAE (Integral of Time Absolute Error), ITSE (Integral of Time Squared Error) has been carried out and presented in Table 2. for both methods of controller design. It is noticable that calculated errors are smaller in the case of AMIGO Based FOPID Controller. As it can been seen the focus in the controller design has been put to the stationary state.

Table 2. Comparison of performance index for servo responses.

Method	IAE	ISE	ITAE	ITSE
AMIGO Based FOPID Controller	18.55	9.846	153.814	33.134
Gain Margin – Phase Margin Controller	19.69	11.18	187.23	46.38

4. CONCLUSION

Performed research leads to the conclusion that fractional - order PI controller designed based on AMIGO method can be used for MIMO processes with dead time. Controller design has been carried out with emphasis to the stationary state more than transition process, taking care about system

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stability. The effectiveness of suggested approach over conventional (integer degree) Gain Margin - Phase Margin controller design is proven by performance indices IAE, ISE, ITAE, ITSE.

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